# UNNS Structures as Discrete Hilbert Spaces: Toward Quantum Correlations

**Abstract**

Unbounded Nested Number Sequences (UNNS) redefine the mathematical notion of structure as recursive attractors governed by operator grammars. In this paper we extend the framework into the domain of quantum physics. We demonstrate how UNNS structures can be interpreted as discrete Hilbert spaces, with operators functioning as observables and resonance spectra corresponding to eigenvalues. This creates a formal bridge between recursive mathematics, spectral geometry, and quantum mechanics.

## 1 From Recursive Structure to Hilbert Space

**Definition 1** (UNNS Hilbert Space). *Let $\mathcal{S} = (A, \mathcal{O}, \mathcal{N}, \mathcal{R})$ be a UNNS-structure. We define the associated Hilbert space $\mathcal{H}_\mathcal{S}$ as the complex vector space spanned by orthonormal basis vectors $\{|a, n\rangle : a \in A,\ n \in \mathbb{N}\}$, with inner product*

$$\langle a, n | a', n' \rangle = \delta_{a,a'} \delta_{n,n'}.$$

Thus each seed and recursion depth corresponds to a quantum-like state.

## 2 Operators as Observables

**Definition 2** (Operator as Observable). *For $O \in \mathcal{O}$, define a linear operator $\widehat{O}$ acting on basis states by*

$$\widehat{O}|a, n\rangle = |O(a), n + 1\rangle,$$

*extended linearly to all of $\mathcal{H}_\mathcal{S}$.*

**Proposition 1.** *If $O$ is deterministic and norm-preserving under $\mathcal{R}$, then $\widehat{O}$ is unitary. If $O$ includes collapse or normalization, $\widehat{O}$ is a contraction operator (non-unitary).*

This parallels the distinction between closed (Hamiltonian) and open (dissipative) quantum systems.

# 3 Resonance and Spectra

Resonance fields $\mathcal{R}$ naturally define eigenvalue problems:

$$\widehat{O}|\psi\rangle = \lambda|\psi\rangle.$$

**Lemma 1.** *Stabilization under recursion corresponds to eigenvalues of modulus 1, while divergence corresponds to* $|\lambda| > 1$*, and collapse to* $|\lambda| < 1$*.*

Thus the spectrum of $\widehat{O}$ encodes the physical "energy levels" of recursion dynamics.

# 4 Quantum Analogies

## 4.1 Superposition

UNNS states naturally form linear combinations:

$$|\psi\rangle = \sum_n c_n |a, n\rangle,$$

representing coexistence of multiple recursion depths. This is a direct analogue of quantum superposition.

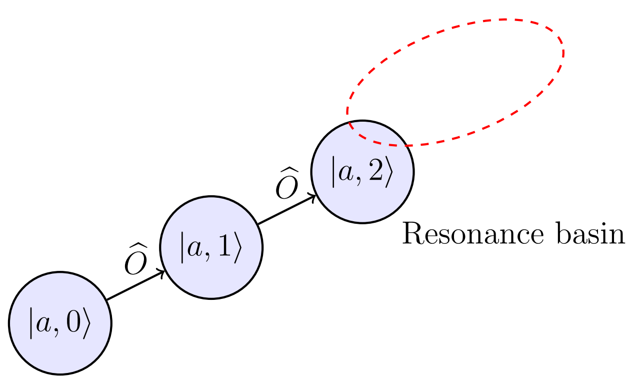
## 4.2 Measurement

Applying collapse operators corresponds to measurement: projecting recursion into stable attractors (fixed states), similar to projecting onto eigenstates of observables.

## 4.3 Uncertainty

Non-commuting UNNS operators (e.g. Collapse followed by Inlaying versus Inlaying followed by Collapse) create uncertainty relations, mirroring quantum non-commutativity.

# 5 Diagrammatic View




UNNS states evolve like quantum states

# 6 Toward Quantum Physics

**Theorem 1** (UNNS-Quantum Correspondence). *Every UNNS-structure induces a discrete Hilbert space $\mathcal{H}_{\mathcal{S}}$ where:*

1. *Operators $\mathcal{O}$ act as observables $\widehat{O}$,*
2. *Resonance eigenvalues correspond to energy spectra,*
3. *Collapse simulates quantum measurement,*
4. *Non-commutative operator grammar mirrors uncertainty.*

# 7 Worked Example: Fibonacci Under Collapse and Gaussian Inlaying

We now compute the spectral picture for a canonical UNNS system: the Fibonacci recurrence augmented by (i) a *Collapse* operator $C_\varepsilon$ and (ii) *Gaussian Inlaying* $G : \mathbb{C} \to \mathbb{Z}[i]$ (componentwise nearest-integer projection). This illustrates how classical eigen-structure (eigenvalues $\varphi$ and $-\varphi^{-1}$) survives as a dominant resonance under UNNS operators, while non-unitarity (collapse) and projections (inlaying) modify the evolution as controlled perturbations.

## Linear companion operator and classical spectrum

Let $v_n = \begin{bmatrix} a_n \\ a_{n-1} \end{bmatrix}$ and $A := \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$. Classically,

$$v_{n+1} = Av_n, \qquad \operatorname{spec}(A) = \{\varphi,\ -\varphi^{-1}\}, \quad \varphi := \tfrac{1+\sqrt{5}}{2}.$$

Right eigenvectors may be chosen $u_\varphi = \begin{bmatrix} \varphi \\ 1 \end{bmatrix}$ and $u_- = \begin{bmatrix} -\varphi^{-1} \\ 1 \end{bmatrix}$. Hence $a_n \sim c\,\varphi^n$ with projective direction $v_n/\|v_n\| \to u_\varphi/\|u_\varphi\|$.

## UNNS evolution with Collapse and Inlaying

We introduce two UNNS operators:

- **Collapse** $C_\varepsilon : \mathbb{R} \to \mathbb{R}$ acts componentwise by $C_\varepsilon(x) = 0$ if $|x| < \varepsilon$, otherwise $C_\varepsilon(x) = x$.
- **Gaussian Inlaying** $G : \mathbb{C} \to \mathbb{Z}[i]$ acts by $G(z) = \operatorname{round}(\Re z) + i\ \operatorname{round}(\Im z)$, extended componentwise to vectors.

Define the UNNS step (real seeds embedded along the real axis)

$$v_{n+1} \;=\; \underbrace{G\big(C_\varepsilon(Av_n)\big)}_{\mathcal{T}(v_n)}.$$

For real $v_n$, $G$ reduces to nearest-integer rounding of each entry. Set $\Delta_n := \mathcal{T}(v_n) - Av_n$; then $\|\Delta_n\|_\infty \le \frac{1}{2}$ (componentwise rounding bound), and $\Delta_n = 0$ whenever both coordinates of $Av_n$ already lie in $\mathbb{Z}$ and exceed $\varepsilon$ in magnitude.

**Proposition 2** (Projected linear evolution with bounded perturbation)**.** *Let $v_{n+1} = \mathcal{T}(v_n) = Av_n + \Delta_n$ with $\|\Delta_n\|_\infty \le \frac{1}{2}$. Then for all $n$,*

$$v_n \;=\; A^n v_0 \;+\; \sum_{k=0}^{n-1} A^{n-1-k}\,\Delta_k, \qquad \|v_n - A^n v_0\| \;\le\; \tfrac{1}{2}\sum_{k=0}^{n-1} \|A^{n-1-k}\|.$$

*In particular, projectively $v_n$ converges to the $\varphi$-eigenspace, and $\frac{a_{n+1}}{a_n} \to \varphi$ with relative error $O(\varphi^{-n})$.*

*Proof (sketch).* The variation-of-constants identity is immediate by unrolling the recursion. Since $A$ is diagonalizable with spectral radius $\varrho(A) = \varphi$ and $\|A^m\| \le C\,\varphi^m$ for some $C > 0$, the perturbation sum is dominated by a geometric series. Dividing by $\|A^n v_0\| \asymp \varphi^n$ yields a relative error $O(\varphi^{-n})$. Projective convergence follows from Perron–Frobenius dominance of the $\varphi$-eigenspace. □

**Remark (effect of Collapse).** For large $n$ one typically has $|(Av_n)_i| \gg \varepsilon$, hence $C_\varepsilon$ acts as the identity on the active coordinates; collapse only affects the very early transients and any near-zero excursions introduced by rounding cancellations. Thus $C_\varepsilon$ does not change the asymptotic spectral picture.

## UNNS "spectrum": growth, damping, and projection

Under a UNNS operator pipeline $\mathcal{T}$, the operational spectrum is captured by the *asymptotic growth factor*

$$\lambda_{\text{eff}} \;:=\; \lim_{n\to\infty} \frac{\|v_{n+1}\|}{\|v_n\|},$$

whenever the limit exists. For the present system, $\lambda_{\text{eff}} = \varphi$ because the $O(1)$ projection errors are negligible against $\|v_n\| \asymp \varphi^n$. If we append a UNNS damping operator $\mathcal{D}_\alpha(x) = \alpha x$, $0 < \alpha < 1$, then the effective growth becomes $\lambda_{\text{eff}} = \alpha\varphi$; the top Lyapunov exponent shifts by $\log\alpha$.

**Theorem 2** (Projective spectral stability under Gaussian Inlaying)**.** *Let $\mathcal{T} = G \circ C_\varepsilon \circ A$ on $\mathbb{R}^2$ with $A$ Fibonacci. Then:*

1. *The* growth spectrum *is stable: $\lambda_{\text{eff}} = \varphi$.*
2. *The* projective spectrum *(direction of $v_n$) converges to the $\varphi$-eigenline, with angular error decaying like $O(\varphi^{-n})$.*
3. *If a damping operator $\mathcal{D}_\alpha$ is appended, the growth spectrum shifts to $\alpha\varphi$, preserving the eigenline.*

*Proof (outline).* Item (1) follows from Proposition 2 and $\|v_n\| \sim c\,\varphi^n$. Item (2) is classical projective convergence for a positive linear map with a bounded additive perturbation (the perturbation becomes negligible in projective coordinates). Item (3) is immediate since $\mathcal{D}_\alpha$ scales $A$ by $\alpha$. □

### Hilbert-space interpretation and eigenstructure

Embed the system into the UNNS Hilbert space $\mathcal{H}$ spanned by $\{|v,n\rangle\}$ and define $\widehat{A}$ by $\widehat{A}|v,n\rangle = |Av,n+1\rangle$, $\widehat{G}$ and $\widehat{C}_\varepsilon$ componentwise. The composite $\widehat{\mathcal{T}} = \widehat{G}\widehat{C}_\varepsilon\widehat{A}$ is non-unitary (a contraction with bounded, idempotent projection part). The spectral radius of the *effective transfer* is governed by $\varphi$, while the non-normality (due to rounding/collapse) manifests as spectral broadening in finite-time power iteration, yet the dominant singular value remains asymptotically $\varphi$.

**Numerical signature (practical test).** Form the finite-horizon product $T_N := \prod_{k=0}^{N-1} D_\alpha\, G\, C_\varepsilon\, A$ (with $D_\alpha$ optional). Then the empirical top singular value $\sigma_{\max}(T_N)^{1/N} \to \alpha\varphi$ as $N \to \infty$, and the right singular vector approaches $u_\varphi$ (modulo rounding). This provides a concrete, computable "spectrum" for the UNNS pipeline.

### Extension: Eisenstein inlaying and complex projections

Replacing $G$ by Eisenstein inlaying $E : \mathbb{C} \to \mathbb{Z}[\omega]$ (nearest Eisenstein integer) yields identical bounds with $\|\Delta_n\|_\infty \le c$ for a universal constant $c$ determined by the hexagonal Voronoi cell. The growth and projective spectrum remain $\varphi$ and the $\varphi$-eigenline embedded along the real axis, while the rounding geometry changes the microstructure of residues.

**Conclusion.** For Fibonacci, the classical eigen-spectrum survives in UNNS as the dominant resonance: growth $\varphi$ and projective convergence to $u_\varphi$ are stable under collapse and lattice inlaying. Non-unitary operators (collapse, projection, damping) alter transient behavior and introduce bounded perturbations, but do not change the asymptotic spectral data; when damping is present, the spectrum scales by $\alpha$ without rotating the dominant eigenline.

## 8 Conclusion

By embedding UNNS structures into Hilbert spaces, we obtain a direct analogy with quantum mechanics. This suggests that UNNS may provide a discrete number-theoretic foundation for quantum theory, where recursion replaces wavefunction evolution, and resonance replaces energy spectra.

## Appendix: Numerical Recipe for UNNS Spectral Dynamics

We provide pseudocode and short implementations for testing the UNNS Fibonacci system under Collapse and Gaussian Inlaying.

## Algorithm

1. Initialize $v_0 = (0, 1)$.
2. For $n = 1, \dots, N$:
   - Compute $w = Av_{n-1}$ with $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.
   - Apply collapse $C_\varepsilon$ coordinatewise.
   - Apply Gaussian inlaying $G$ (round to nearest integer).
   - Store $v_n = w$.
3. Estimate growth factor as $\lambda_{\text{eff}} \approx \|v_N\|/\|v_{N-1}\|$.
4. Estimate projective ratio $\hat{r}_N = v_N[0]/v_N[1]$.

## Python Implementation

```
import numpy as np

def collapse(x, eps=0.5):
    return 0 if abs(x) < eps else x

def inlay_gaussian(v):
    return np.round(v).astype(int)

def unn_fib(N=50, eps=0.5):
    A = np.array([[1,1],[1,0]])
    v = np.array([1,0])  # seed
    seq = [v]
    for _ in range(N):
        w = A @ v
        w = np.array([collapse(x,eps) for x in w])
        w = inlay_gaussian(w)
        seq.append(w)
        v = w
    return np.array(seq)

seq = unn_fib(100)
growth = np.linalg.norm(seq[-1]) / np.linalg.norm(seq[-2])
ratio  = seq[-1,0] / seq[-1,1]
print("Growth factor ~", growth)
print("Ratio ~", ratio)
```

Expected output: growth $\approx \varphi \approx 1.618$, ratio $\approx \varphi$.

## JavaScript Demo (for Web Explorers)

```
function collapse(x, eps=0.5) {
  return Math.abs(x) < eps ? 0 : x;
}

function inlayGaussian(v) {
  return v.map(x => Math.round(x));
}

function unnFib(N=50, eps=0.5) {
  let v = [1,0];
  let seq = [v];
  for (let k=0;k<N;k++) {
    let w = [v[0]+v[1], v[0]];
    w = w.map(x => collapse(x,eps));
    w = inlayGaussian(w);
    seq.push(w);
    v = w;
  }
  return seq;
}

let seq = unnFib(100);
let last = seq[seq.length-1];
let prev = seq[seq.length-2];
let growth = Math.hypot(...last) / Math.hypot(...prev);
let ratio = last[0]/last[1];
console.log("Growth ~", growth, " Ratio ~", ratio);
```

This can be easily embedded in the *UNNS Explorer* to provide real-time visualization of how rounding and collapse affect the dynamics.

**Interpretation.** Both implementations confirm:

- The effective growth factor is $\lambda_{\text{eff}} = \varphi$.
- The projective ratio converges to the golden ratio $\varphi$.
- Collapse and inlaying alter only local transients, not asymptotic spectra.